In this section, authors delve into a real-life application of the OMERF method, conducting an analysis with the objective of comparing its performance with the models delineated in Section 4.

The data employed in the study concern 15-year-old students from Italy who completed the PISA 2022 survey questionnaire. The dataset comprises information on 10,552 students across 340 schools. After filtering for complete cases and schools with more than 10 students, the processed dataset consists of 7,639 observations, representing students enrolled in 293 schools. For model training and evaluation, a random sample comprising 80% of the observations is designated as the training set, while the remaining 20% constitutes the test set.

The objective of this case study is to evaluate the performance of the OMERF in predicting students' mathematical performance. In accordance with the threshold established by the OECD, the output variable is categorized into three ordinal levels: the lowest level encompasses students classified in levels 1 or 2, while the highest level includes those achieving levels 5 or 6, with the remaining students falling within the intermediate class. Demographic factors, such as gender and immigrant status, alongside educational indicators like grade level and family background - including parents' highest education level and ESCS - are considered as covariates for prediction. Additionally, factors related to home environment, such as access to and quality of internet services and video games, as well as the number of hours spent on homework per week, and perceived support from both teachers and parents, are incorporated into the analysis. School climate aspects, including the perception of school risk, sense of belonging, and experiences of bullying, are also used. Furthermore, self-perception attributes like cooperation, perseverance, assertiveness, empathy, emotional control, stress resistance, and curiosity are included as relevant predictor variables. A detailed description of these variables, along with corresponding descriptive statistics, is provided in Table X.

Table X: Variable description and descriptive statistics

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Variable name** | **Variable description** | **Variable type** | **Distribution** | **Mean** | **SD** | **Min** | **Max** |
| stud\_ID | Student identification number | factor | 7639 unique values |  |  |  |  |
| school\_ID | School identification number | factor | 393 levels |  |  |  |  |
| mate3 | Output variable, mathematics test score | factor | 1: 3634, 2: 3401, 3: 604 |  |  |  |  |
| gender | Student gender | factor | 1: 4041, 0: 3598 |  |  |  |  |
| immig | Student immigration status | factor | 0: 6835, 1: 586, 2: 218 |  |  |  |  |
| grade | Student grade attended | factor | 9: 739, 10: 6485 , 11: 415 |  |  |  |  |
| video\_games | Frequency of use at home of video or online games | numeric |  | 3,27 | 1,59 | 1,00 | 6,00 |
| internet\_quality | Quality of access to ICT | numeric |  | -0,18 | 0,83 | -2,80 | 2,89 |
| internet\_availability | ICT availability outside of school | numeric |  | 5,70 | 0,91 | 0,00 | 6,00 |
| SCHRISK | School safety risks | numeric |  | 0,01 | 0,82 | -0,46 | 3,05 |
| BULLIED | Being bullied | numeric |  | -0,47 | 0,87 | -1,23 | 4,69 |
| BELONG | Sense of belonging | numeric |  | 0,01 | 0,89 | -3,26 | 2,78 |
| COOPAGR | Cooperation (agreement) | numeric |  | 0,12 | 1,00 | -5,24 | 6,13 |
| TEACHSUP | Teachers support | numeric |  | -0,20 | 1,11 | -2,91 | 1,56 |
| FAMSUP | Family support | numeric |  | -0,02 | 0,93 | -3,01 | 1,96 |
| PERSEVAGR | Perseverance (agreement) | numeric |  | 0,07 | 0,98 | -5,91 | 4,89 |
| ASSERAGR | Assertiveness (agreement) | numeric |  | -0,03 | 1,01 | -8,23 | 7,23 |
| EMPATAGR | Empathy (agreement) | numeric |  | 0,01 | 0,99 | -6,46 | 4,69 |
| EMOCOAGR | Emotional control (agreement) | numeric |  | -0,09 | 0,98 | -5,17 | 5,58 |
| STRESAGR | Stress resistence (agreement) | numeric |  | -0,18 | 1,00 | -5,26 | 5,49 |
| CURIOAGR | Curiosity (agreement) | numeric |  | 0,09 | 0,96 | -4,95 | 4,18 |
| study\_time | Total time for all homework in all subjects per week | numeric |  | 3,50 | 1,51 | 1,00 | 6,00 |
| HISCED | Highest level of education of parents | numeric |  | 7,03 | 2,14 | 1,00 | 10,00 |
| ESCS | Escs | numeric |  | -0,01 | 0,87 | -3,23 | 2,78 |

**Case study - Results**

In Table Y, the performance indices previously explained and used for the simulation study are presented to assess the predictive power of the OMERF algorithm.

[inserire tabella indici]

From these values, it can be noticed that the CLMM and OMERF algorithms shows comparable performance indices and outperform other algorithms across all considered metrics. In particular [inserire qualche esempio]. These results underscore the importance of accounting for the hierarchical structure of the data, as evidenced by the performance of models that consider the random effects. Moreover, the OMERF algorithm allows to interpret the results according to the random forest tools. Variable importance analysis, depicted in Figure Z, reveals ESCS as the most influential predictor, consistent with existing literature. Additionally, partial plots, illustrated in Figure W, enable examination of the relationship between predictors and the output variable. Some variables like [inserire alcune non lineari] present a non linear relationship that OMERF, in contrast with CLMM, is able to capture. ESCS demonstrates a quasi-linear relationship with mathematical performance, potentially explaining the comparable performance of OMERF and CLMM, as linear models can adequately capture such relationships.

[inserire variable importance plot]

[inserire partial plot]

[se funziona inserire le performance del modello senza ESCS]